Polyhedral Approach for the Vertex Separator Problem

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Abstract:

In a connected undirected graph $G = (V, E)$, a separator is a subset of vertices (or edges) whose removal disconnects G. Finding a balanced minimum-weight separator in a graph is relevant in many problems.

In the field of networks and telecom, such a separator is seen as a bottleneck when a graph represents the network. In the field of graph algorithms, the computation of balanced small-sized separators is very useful, especially for divide-and-conquer algorithms. In the field of automatic treatment of written language, separators are used to split large sparse graphs of terms. The vertices of the graph are the terms of the considered domain. The edges of the graph stand for syntaxical relations between terms. The aim is to provide thematical classes from textual corpus. Hierachical classification is used but some classes can be too large to be interpreted by users. Thus, they could be factorized using separators [4]. In the field of bioinformatics and computational biology, separators are wanted in grid graphs providing a simplified representation of proteins.

Formally, the vertex separator problem can be stated as follows. Given $G = (V, E)$ a connected undirected graph and a positive integer $\beta(|V|)$, the vertex separator problem is to find a partition of V into three classes A, B, C such that there is no edge between A and B, $\max\{|A|, |B|\} \leq \beta(n)$ and $\sum_{i \in C} w_i$ is minimum (where w_i is a cost associated to the vertex $i \in V$).

The vertex separator problem is NP-hard [1]. In 2005, Egon Balas and Cid De Souza provided the first polyhedral study of the vertex separator problem ([2],[3]). In our work, we consider the vertex separator problem from a polyhedral point of view, in the particular case when $w_i = 1$, $\forall i \in V$. We introduce a new class of valid inequalities for the associated polyhedron. Those inequalities give us a good lower bound for the optimal solution. We present successful computational experiments.

Bibliography

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